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ABSTRACT

Pre-algebra students (N=98) were asked to sort a collection of word problems into groups of problems which they thought were mathematically related. On the basis of these sorts, four dimensions of problem-relatedness were identified: (1) mathematical structure, (2) contextual details, (3) question form, and (4) pseudostructure (quantity measured such as age or weight). Students who sorted the problems on the basis of mathematical structure tended to be high in mathematical ability. Correlations between preference scores for each of the four dimensions were computed and examined. (SD)

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Student Classification of Mathematical Word Problems: Bases for Classification and Relationships to Problem Solving and Other Abilities

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Abstract

On two occasions, once before and once after solving the problems, a group of 98 eighth grade students sorted a collection of mathematical word problems into groups that they thought were mathematically related. On the basis of their criteria and sorts, four dimensions were identified: (a) mathematical structure, (b) contextual details, (c) question form, and (d) pseudostructure (quantity measured, such as age or weight). In general, students perceived the structure dimension as independent of the other three dimensions but tended to associate the context, question, and pseudostructure dimensions in their sorts.

Students who tended to classify problems on the basis of mathematical structure tended to be high in mathematical ability, as measured by nonverbal IQ, computational ability, mathematics concepts knowledge, abstract reasoning ability, or problem-solving ability. Students who tended to classify problems on the basis of contextual details tended to be low in mathematical ability.

The strong positive relationship between a student's tendency to sort according to mathematical structure and his problem-solving ability was subjected to partial correlation analysis. When the effects of verbal and nonverbal IQ, concepts knowledge, and computational ability were simultaneously controlled, the relationship remained significant ($p<.01$).

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Student Classification of Mathematical Word Problems: Bases for Classification and Relationships to Problem-Solving and Other Abilities[1]

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In his classic treatise on mathematical problem solving, How to Solve It, Polya (1957) has suggested a series of heuristic precepts that may be useful in the solution of a mathematical problem. For example, Polya has suggested that, when one is devising a plan for solving a problem, it is often useful to think of a "related" problem:

> The materials necessary for solving a mathematical problem are certain relevant items of our formerly acquired mathematical knowledge, as formerly solved problems, or formerly proved theorems. Thus, it is often appropriate to start the work with the question: Do you know a related problem? (p. 9)

Indeed, it is reasonable to assume that many mathematics teachers suggest this advice to their students. But how do students respond to the suggestion? If they consider related problems, are the problems related in a mathematically useful way? If the problems are not so related, then why did the students recall them? How do students interpret the word "related"? How do students incorporate into their memories, for later access, the large number of problems they have encountered?

The answers to these questions may be fundamental to an understanding of the psychological implications of Polya's logical suggestion to think of a related problem. In fact, the answers

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may help to provide increased understanding of the modes of information processing and retrieval that are helpful in mathematical problem solving.

Consider a person's mathematical problem-solving experience. The problems he has solved previously have been classified in a certain way. Each new problem is not encountered as a discrete unit, but is interpreted on the basis of the person's previous experience. A person's ability to make good use of Polya's "related problem" heuristic advice is, at least partially, a function of the criteria he has used in forming his categories. The criteria reveal the dimensions along which the person views problems as similar. The similarity dimensions active in a person's perceptions of relatedness help to structure his problem-solving experience.

In recent years, there has been a growing interest in the teaching of mathematical problem solving. How a student perceives mathematical word problems, how he internalizes them and organizes them for easy access, and how he views relatedness among problems are all fundamental data that should be considered in the design of any instructional program in problem solving. Yet, there has been a dearth of research on these topics.

Only one researcher has reported evidence that possibly links the nature of a student's perceptions of relationships among problems and his ability to solve the problems. The Russian psychologist V. A. Krutetskii (1969, 1976) discerned both qualitative and quantitative differences between the memories of good problem solvers and the memories of poor problem solvers. Specifically, he found that good problem solvers tended to recall the structure.

of a problem for a long time, whereas poor problem solvers tended to remember, if anything at all, the details of the problem statement. His research pointed to differences between good and poor problem solvers' perceptions of important element within a problem.

Purpose

The implicit purpose of this study was to examine the underlying structure that students impose on their problem-solving experience. By observing students' criteria for judgements of problem relatedness, the investigator sought to identify the problem similarity dimensions that structured students' perceptions of mathematical problems. After the dimensions were identified, the investigator sought to examine the relationship between differences in students' perceptions of problem similarity and their mathematical ability.

The specific purposes of this investigation were: (1) to examine the problem similarity dimensions perceived by a student when classifying mathematical word problems as "related," and (2) to examine the relationship between a student's perceptions of problem similarity and his performance on a variety of individual difference measures.

Procedure

Method

The principle technique used to examine students' perceptions of problem similarity was a card-sorting task; the card-sorting procedure is quite common in studies of concept formation and seemed appropriate to measure the concept of problem relatedness.

Using a set of 24 mathematical word problems, each on a separate index card, 98 eighth grade students formed groups of problems that they thought were mathematically related. The problems used in this task were carefully constructed so that they varied systematically along the two dimensions identified by Krutetskii — the structure of the problems and the contextual details of the problems. Students were asked to give a reason for each category formed.

The card-sorting task was administered on two occasions; once before and once after the students solved the problems. In the time between the two card sorts, each student attempted to solve each problem and participated in a discussion of the correct solution for each problem.

The second purpose of the study was accomplished by gathering ability data throughout the school year. The students were assessed using measures of verbal and nonverbal intelligence; numerical, verbal and abstract reasoning abilities; mathematics concepts knowledge and computation ability; predicted success in algebra; field dependence/independence; syntactic and semantic abilities and preferences; and mathematical problem-solving ability. Analyses of the data were directed at the identification of relationships between a student's performance on the ability measures and his perceptions of problem relatedness.

Scoring Procedures

For each subject, for each sort, and for each dimension, two scores were obtained: a dimensional association score and a "pure" category score. The dimensional association score was obtained by counting the instances where two problems, related along

the dimension, were placed in the same category. The "pure" category score was obtained by counting the instances of categories which contained two or more problems related along one dimension and contained no unrelated problems. For each subject and for each dimension, the four scores were summed to obtain the dimensional preference score (DPS).

Results

Analyses of students' criteria for categories formed in the card-sorting task indicated that they were associating problems along four problem similarity dimensions. The structure and context dimensions were evident in their categories, and two new dimensions were identified — question form and problem pseudostructure. Many students appeared to be associating problems because the problems contained the same question form, such as "how many?" or "how much?". Students also appeared to be associating problems because they involved the same quantity, such as age, weight, or time. This latter characteristic of problems was called pseudostructure since it sometimes takes on the appearance of the mathematical structure dimension.

Pearson product-moment correlation coefficients were computed for students' dimensional preference scores. The correlation analysis revealed: (1) a significant negative relationship ($r=-.7278$; $p<.001$) between structure DPS and context DPS, (2) a significant negative relationship ($r=-.4860$, $p<.001$) between structure DPS and pseudostructure DPS, (3) a significant positive relationship ($r=.2200$; $p<.05$) between question DPS and pseudostructure DPS, and (4) a significant positive relationship

($r=.6440$; $p<.001$) between context DPS and pseudostructure DPS. The other correlations were not significant.

Analysis of the data related to individual difference variables revealed a significant positive association between structure DPS and performance on measures of verbal and nonverbal intelligence, concepts knowledge, computational and numerical abilities, abstract reasoning and verbal abilities, syntactic ability, syntactic preference, predicted success in algebra, and problem-solving ability. Furthermore, context DPS was significantly negatively associated with all of the above and also with field independence. Pseudostructure DPS was significantly negatively related to nonverbal intelligence, computational and numerical abilities, abstract reasoning and verbal abilities, syntactic ability, and predicted sucess in algebra.

The significant relationship ($r=.5390$; $p<.001$) between structure DPS and problem-solving ability was subjected to partial correlation analysis. When the effects of verbal IQ, nonverbal IQ, mathematics concepts knowledge, and computational ability were simultaneously controlled, the relationship between structure DPS and problem-solving ability remained significant ($r=.2769$; $p<.01$).

Discussion

One major focus of this investigation was the identification of the problem similarity dimensions perceived by students. Four dimensions were identified: (1) the mathematical structure of the problems, (2) the contextual details of the problem statements, (3) the question asked in the problems, and (4) the pseudo-

structure of the problems. The dimensions are similar to those identified by Chartoff (1976).

Both studies identified mathematical structure, contextual details, and question form as important dimensions evident in students' judgements of similarity among problems. Chartoff's fourth dimension, comparison with a generic problem of the same type, was not evident in this study since there were no generalizations or specializations in the problem set. The pseudostructure dimension, identified in this study, was subsumed by the context dimension in Chartoff's study.

Since the card-sorting task was designed with problems varying systematically along the structure and context dimensions, the subsequent emergence of the question and pseudostructure dimensions presents difficulty when interpreting results. For example, seven pairs of problems were related along both the structure and context dimensions. The overlap among the dimensions within the problem set complicates analysis of students' perceptions of independence among dimensions.

Despite the complications, some conclusions are possible. Students appeared to perceive the structure dimension as independent of the other three dimensions. The significant negative correlations for structure with context and with pseudostructure support such a conclusion. Furthermore, the nonsignificant correlation for structure with question, despite the substantial overlap for these dimensions within the problem set, suggests the independence of the two dimensions.

Conclusions regarding students' perceptions of the relationships among the context, question, and pseudostructure dimensions

are more tentative. The positive association between context and pseudostructure may be an artifact of the design of the problem set, since five pairs of problems were related along both dimensions. The positive association between question and pseudostructure may also be an artifact of the design, reflecting a tendency to ask the same question in problems involving the same quantity. The non-significant association between context and question may be a result of both the strong internal overlap for structure and question and the significant negative correlation for structure and context.

Dimensional independence, as perceived by students, deserves more careful attention. A follow-up study is planned, in which the problem set will be revised so that the problems vary systematically along all four dimensions and so that the internal overlap among dimensions is substantially reduced.

The second major focus of this investigation was the relationship between students' perceptions of problem similarity and their performance on individual difference measures. The findings of the study must be interpreted and generalized within the limitations of the populations studied, the representativeness of the CST problem set, and the appropriateness of the individual difference measures.

Since structure DPS is a measure of the degree to which students perceive the mathematical structure of word problems, the results are generally supportive of the findings of Krutetskii (1976). He reported that mathematically capable students, so designated by teacher judgement, are characterized by an ability

to "grasp" the structure of a problem. The results of this study suggest that, whether one uses nonverbal IQ, computational ability, numerical ability, concepts knowledge, abstract reasoning ability, or problem-solving ability to determine a student's mathematical ability, mathematically capable students both perceive the structure of problems and relate problems on the basis of that perception. Moreover, the results support Krutetskii's conclusion that the least capable students tend to concentrate on the details of a problem's statement.

Problem-solving ability was of particular interest in this study. The significant positive relationship between problem-solving ability and structure DPS was analyzed using partial correlations. The analysis revealed that the relationship remained significant even when the effects of IQ, computational ability, and concepts knowledge were simultaneously controlled. The results suggest that structure DPS may be a measure of an independent mathematical ability, not entirely composed of general mathematical ability components, that is significantly related to problem-solving ability. Further research is needed to replicate the findings and examine the implications.

Also suggestive of further research are (1) the strong positive association ($r=.6930$; $p<.001$) between structure DPS and predicted success in algebra, (2) the strong negative association ($r=-.4292$; $p<.001$) between context DPS and problem-solving ability, and (3) the strong negative association ($r=-.6107$; $p<.001$). between context DPS and predicted success in algebra. Investigations into the potential diagnostic and predictive value of the card-sorting task, or some appropriate modification, would seem appropriate.

One goal of such investigations should be the development of instructional strategies and materials designed to teach low ability students to disregard the context, question, and pseudostructure dimensions. In general, the results of this study suggest that more attention must be given to identifying and promoting useful information-processing behavior and to detecting and discouraging less useful modes of processing problem-solving information.

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